Asian Options with Cost Of Carry Zero

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Abstract

The Turnbull and Wakeman (1991) formula is a well known formula for continuous Arithmetic average rate options. Turnbull and Wakeman originally only developed their formula for Asian options when cost-of-carry is different from zero. In many commodity and energy markets where Asian options frequently trade the average is typically based on futures or forward prices, that is cost-of-carry on the underlying asset is zero[1]. Many people have contacted me over the years to ask me for how to extend the Turnbull and Wakeman (1991) formula to also hold in the case of cost-of-carry zero. This quick note gives you the solution.

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Cost-of-Carry Zero Solution

In the case of Asian options when cost of carry is zero the original Turnbull and Wakeman formulas do not hold and must be modified. If we assume the arithmetic average is approximately lognormally distributed all we need to value an Asian futures option is to adjust the volatility of the Black-76 formula. This entails replacing the futures volatility σ with the volatility of the average on the futures σ_A:

$$c_A \approx e^{-rT}[FN(d_1) - XN(d_2)], \tag{1}$$

$$p_A \approx e^{-rT}[XN(-d_2) - FN(-d_1))], \tag{2}$$

*This solution is based on some calculations I did on May 16 1999.

[1]Options on stocks can naturally also have cost-of-carry zero if the continuous dividend yield equal to the risk free rate, the extension given in this note can then naturally also be used.

where T is the time to maturity, r is the risk-free rate, F is the futures price and X is the strike price.

$$d_1 = \frac{\ln(F/X) + T\sigma_A^2/2}{\sigma_A\sqrt{T}}, \qquad d_2 = d_1 - \sigma_A\sqrt{T},$$

where[2]

$$\sigma_A = \sqrt{\frac{\ln(M)}{T}}, \qquad M = \frac{2e^{\sigma^2 T} - 2e^{\sigma^2\tau}[1 + \sigma^2(T-\tau)]}{\sigma^4(T-\tau)^2},$$

where τ is the time to the beginning of the average period and σ is the volatility of the futures contract. If the option is into the average period the strike price must be replaced by $\hat{X}$ and the option value must be multiplied by $\frac{T}{T_2}$, where

$$\hat{X} = X\frac{T_2}{T} - F_A\frac{(T_2 - T)}{T},$$

where T_2 is the original time in the average period and F_A is the average futures price during the realized or observed time period $T_2 - T$.

If $\hat{X}$ should be negative the call option will for sure be exercised at maturity and the value becomes the discounted value of the expected average at maturity $E_Q[A]$ minus the strike price: $E_Q[A] - X$. The expected average is equal to

$$E_Q[A] = \frac{F_A(T_2 - T)}{T_2} + F\frac{T}{T_2}.$$

For a put the value will be 0 if $\hat{X}$ should be negative. This is basically the Turnbull-Wakeman formula extended to Asian options on futures (cost-of-carry zero).

Conclusion

We have here extended the Turnbull and Wakeman formula to also hold for options in the case of zero-cost-of carry.

I must however say it is a mystery to me why so many people in practice are interested in continuous average rate options formulas when all Asian options in practice are discrete type, for more on this see for example Haug (2006b) and also the 2nd edition of "The Complete Guide To Option Pricing Formulas".

[2]It is only the second moment we give here, the first moment is 1 with cost-of-carry zero.

References

Haug, E. G. (2006a): *The Complete Guide To Option Pricing Formulas, 2nd Edition*. McGraw-Hill, New York.

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Turnbull, S. M., and L. M. Wakeman (1991): "A Quick Algorithm for Pricing European Average Options," *Journal of Financial and Quantitative Analysis*, 26, 377–389.