New full relativistic escape velocity and new Hubble related equation for the universe

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Abstract: The escape velocity derived from general relativity coincides with the Newtonian one. However, the Newtonian escape velocity can only be a good approximation when $v \ll c$ is sufficient to break free of the gravitational field of a massive body, as it ignores higher-order terms of the relativistic kinetic energy Taylor series expansion. Consequently, it does not work for a gravitational body with a radius at which v is close to c such as a black hole. To address this problem, we revisit the concept of relativistic mass, abandoned by Einstein, and derive what we call a full relativistic escape velocity. This approach leads to a new escape radius, where $v_e = c$ equal to a half of the Schwarzschild radius. Furthermore, we show that one can derive the Friedmann equation for a critical universe from the escape velocity formula from general relativity theory. We also derive a new equation for a flat universe based on our full relativistic escape velocity formula. Our alternative to the Friedmann formula predicts exactly twice the mass density in our (critical) universe as the Friedmann equation after it is calibrated to the observed cosmological redshift. Our full relativistic escape velocity formula also appears more consistent with the uniqueness of the Planck mass (particle) than the general relativity theory: whereas the general relativity theory predicts an escape velocity above c for the Planck mass at a radius equal to the Planck length, our model predicts an escape velocity c in this case. © *2021 Physics Essays Publication.*
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Résumé: La vitesse d'échappement dérivée de la relativité générale coïncide avec celle de Newton. Cependant, la vitesse d'échappement newtonienne ne peut être une bonne approximation que lorsque v ≪ c est suffisant pour se libérer du champ gravitationnel d'un corps massif, car elle ignore les termes d'ordre supérieur de l'expansion relativiste de l'énergie cinétique en série de Taylor. Par conséquent, cela ne fonctionne pas pour un corps gravitationnel avec un rayon auquel v est proche de c, comme un trou noir. Pour résoudre ce problème, nous revisitons le concept de masse relativiste, abandonné par Einstein, et dérivons ce que nous appelons une vitesse d'échappement relativiste complète. Cette approche conduit à un nouveau rayon d'échappement où ve = c égal à la moitié du rayon de Schwarzschild. De plus, nous montrons que l'on peut dériver l'équation de Friedmann pour un univers critique à partir de la formule de vitesse d'échappement de la théorie de la relativité générale. Nous dérivons également une nouvelle équation pour un univers plat basée sur notre formule de vitesse d'échappement relativiste complète. Notre alternative à la formule de Friedmann prédit exactement le double de la densité de masse dans notre univers (critique) comme l'équation de Friedmann après avoir été calibrée sur le décalage vers le rouge cosmologique observé. Notre formule de vitesse d'échappement relativiste complète semble également plus cohérente avec l'unicité de la masse de Planck (particule) que la théorie de la relativité générale: alors que la théorie de la relativité générale prédit une vitesse d'échappement supérieure à c pour la masse de Planck à un rayon égal à la longueur de Planck, notre modèle prédit une vitesse d'échappement c dans ce cas.

Key words: Hubble Constant; Escape Velocity; Schwarzschild Radius; Hubble Radius; Friedmann Equation; Schwarzschild Sphere; Planck Scale.

I. THE NEWTON ESCAPE VELOCITY, THE GENERAL RELATIVITY ESCAPE VELOCITY, AND A NEW FULL RELATIVISTIC ESCAPE VELOCITY

The Schwarzschild radius is simply the radius, at which when a given mass is inside a sphere of this radius, the escape velocity is $v_e = c$. The idea of massive gravity objects, from which not even photons could escape, does not come from Schwarzschild[1,2] or from Einstein's general relativity theory,[3] but indirectly from Newton's[4] theory. Already in 1784, John Michell[5] wrote

> *If the semi-diameter of a sphere of the same density as the Sun were to exceed that of the Sun in the proportion of 500 to 1, a body falling from an infinite height towards it would have acquired at its surface greater velocity than that of light and consequently supposing light to be attracted by the*

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same force in proportion to its vis inertia, with other bodies, all light emitted from such a body would be made to return towards it by its own proper gravity. This hypothesis assumes that gravity influences light in the same way as massive objects.

Pierre-Simon Laplace[6] suggested the same idea in his book published in 1796. The described radius is basically identical to the Schwarzschild radius

$$R_s = \frac{2GM}{c^2}, \tag{1}$$

and the interpretation that light cannot escape the gravity of a body with this radius is very similar to that suggested for black holes. Michell suggested dark stars, from which not even light could escape. The Michell (dark star) radius was rooted in Newtonian mechanics. Indeed, we can set the kinetic energy of the small mass to be equal to the gravitational potential energy to find the escape velocity from Newtonian mechanics

$$\frac{1}{2}mv^2 - G\frac{Mm}{r} = 0. \tag{2}$$

Solving for v, we obtain the well-known formula

$$v = \sqrt{\frac{2GM}{R}}. \tag{3}$$

This equation is called the escape velocity, and the notation v_e is often used. If we now set $v_e=c$ and solve the equation with respect to R, we get

$$R = \frac{2GM}{c^2}. \tag{4}$$

This result is identical to the Schwarzschild radius but calculated from classical Newton's mechanics with no reliance on the general relativity theory or the Schwarzschild solution. The general relativity theory gives exactly the same escape velocity[7] formula. This coincidence has also been noticed by other authors, for example, Chandrasekhar:[8]

By a curious coincidence, the limit R_s discovered by Laplace is exactly the same that general relativity gives for the occurrence of the trapped surface around a spherical mass.

Chandrasekhar and Hawking[9] emphasized the similarity between Newton dark bodies and black holes, not only mathematically but also to a large degree in interpretation.

On the other hand, for example, Loinger[10] claims "The dark body of Michell-Laplace has nothing to do with the relativistic black hole." In several books on the general relativity theory, the escape velocity, even in the context of Schwarzschild black holes, is derived from Newton mechanics. Other authors[11] call for a more rigorous analysis (from the general relativity theory) of the general relativity theory but agree that it comes to the exact same Schwarzschild radius equation as the one derived from Newtonian mechanics. However, several researchers have rightly pointed out that the interpretation of the escape velocity in Newtonian and general relativity pictures can be very different. Clearly, researchers do not agree on the interpretations of the "black hole mathematical framework."

In this paper, we suggest that neither the general relativity theory nor Newtonian mechanics can fully describe the escape velocity, and therefore, they must be incomplete at their foundations. This hypothesis should naturally be discussed further before any final conclusions are made. However, already in this paper, we demonstrate good reasons to think that the standard escape velocity formula is incomplete.

A deeper consideration makes the idea that general relativity should give the same escape velocity as Newtonian mechanics feels a little strange. The Newton solution involves $E_k \approx 1/2mv^2$, which is a good approximation for kinetic energy only when $v \ll c$. Hence, the Newtonian solution cannot be used for finding the radius where the escape velocity is $v_e = c$, even if it gives exactly the same final mathematical result as the Schwarzschild solution. Conversely, we can ask how the Schwarzschild solution from general relativity theory can give exactly the same mathematical result as the Newtonian mechanics if we know that the Newton solution does not hold for v close to c. We will look closer at that point in this paper.

The small mass in the Newtonian formula should also be expected to be relativistic to incorporate Einstein's relativistic kinetic energy. This brings us to relativistic mass. Already in 1899, Lorentz[12] among others suggested that the mass of an object increased when it was moving, but that the effect was different for different directions relative to the observer. In 1903, Abraham[13] introduced terms *longitudinal* and *transverse* mass for moving masses. Thomson[14] in 1904 mentions that mass will increase as it moves, but that this effect would be directionally dependent. The correct relativistic mass formula was given by Lorentz[15] already in 1899 (and also in 1904). He presented two formulas: one for transverse relativistic mass $m_T = m\gamma$ and another for longitudinal mass $m_L = m\gamma^3$. Lorentz's transverse moving mass formula corresponded to what is known today as relativistic mass (for any direction). Einstein likely did not know about the Lorentz 1899 or 1904 paper. In his[16] famous 1905 paper, he introduced a special relativity theory to derive formulas for relativistic mass, on which he was likely incorrect. Einstein had derived the relativistic energy correctly and was the first to introduce it as

$$E = mc^2\gamma. \tag{5}$$

By simply dividing by c^2 on both sides, Einstein would have arrived at the correct relativistic mass. Instead, he followed the "speculative" tradition laid out before him, trying to perform separate derivations of longitudinal mass and transverse mass. In his 1905 paper on relativity theory, he gave the following relativistic mass results:

$$\text{longitudinal mass} = m\gamma^3 = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}^3}. \tag{6}$$

This equation is the same longitudinal mass that Lorentz suggested one year before, but since Einstein did not refer to Lorentz, it is likely that he was not aware of Lorentz's paper. For transverse mass, Einstein suggested

$$\text{transverse mass} = m\gamma^2 = \frac{m}{1-\frac{v^2}{c^2}}. \quad (7)$$

This result is different from the Lorentz transverse mass and today known to be likely incorrect. None of Einstein's relativistic mass formulas correspond to the well-known relativistic mass as we know it today (see, for example, Refs. 17–19)

$$m_r = m\gamma = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}. \quad (8)$$

Some researchers prefer to write this formula in a somewhat different notation

$$m = m_0\gamma = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}, \quad (9)$$

where m_0 is the rest-mass. In 1908, Lewis[20] presented the relativistic mass formula for any direction as we know it today. It is the same as Eqs. (5), (8), and (9).

Furthermore, in 1909, Lewis and Tolman[21] correctly derived the relativistic mass formula from Newtonian mechanics. In 1912, Tolman[22] insisted that the relativistic mass given in Eqs. (8) and (9) was the right and relevant relativistic mass. In 1934, he argued that only the transverse mass $m = m_o\gamma$ makes sense "since this is the quantity that will give momentum multiplied by the velocity of the particle. It is the quantity that is conserved when particles interact by collision."[23] This view was also held by other researcher, such as Vereide[24] in 1921, according to whom the physics community agreed that the relevant moving mass formula was $m_0\gamma$, as only this description of moving mass seemed consistent with principles of conservation of moving masses (and momentum), unlike Bucherer and Abraham and others' suggestions. This debate was, however, in no way fully settled by then, or even now. Several prominent authors kept referring to Einstein's likely incorrect 1905 moving mass formulas. For example, Wien, the 1911 Noble Prize laureate, in 1921 referred to Einstein's transverse mass as $m\gamma^2$ without any critics of it.[25]

Einstein proposed an experimental method to distinguish between the different theoretical ideas of transverse and longitudinal mass in his 1906 paper.[26] He distinguished between the theories of Bucherer and Abraham. He also referred to his own and Lorentz's theories as though they were the same, which is surprising given that Lorentz's theory was different with respect to moving masses, and gave a different transverse mass formula, as pointed out above. Thus, Einstein's deliberations did not seem to explain much in the context of the relativistic mass.

In 1907 Einstein[27] wrote "Thus, a system of moving mass points—taken as a whole—has the more inertia the faster the mass points move relative to each other." Einstein here clearly indicates that at least the inertia of mass is relativistic.

Actually, Max Born in 1920[28] was possibly the first person to coin the name "relativistic mass" for formula (8). Lewis[29] in 1925 used this term for Lorentz's mass formula (correctly, not Einstein's mass formula), and so Lorentz is the inventor of what is commonly known today as the relativistic mass formula.

Max Planck[30] had derived the relativistic momentum in 1906, $p = mv\gamma$, which Einstein[31] mentioned first in 1907, without reference to Planck. Standard theory today typically relies on relativistic momentum instead of relativistic mass, but the relativistic momentum is simply the relativistic mass multiplied by v. Actually, standard momentum $p = mv\gamma$ is likely not valid for the rest mass,[32] which is partly the reason why the four-vector approach is needed in our view. When the mass is at rest, its momentum is zero, so the standard momentum needs to be replaced with the rest-mass energy divided by c, which forms the so-called time component of the four-momentum. The time component is identical to what Haug recently coined Compton momentum when the particle is at rest.[33] The Compton momentum, $p_t = mc\gamma$, is well defined for any velocity from 0 to c ($v < c$), unlike the standard momentum that is not defined for $v = 0$. (One could try to argue that it is zero in that case, but then again in the fourth-momentum framework it is replaced by the rest-mass energy divided by c.) An in-depth discussion of this problem is beyond the remit of this paper.

Many researchers[34–37] strongly criticize using the relativistic mass, calling it a mathematical artifact. Adler has claimed:

Anyone who has tried to teach special relativity using the four-vector space-time approach knows relativistic mass and four-vectors make for an ill-conceived marriage. In fact, most of the recent criticism of relativistic mass is presented in the context of the four-vector formulation of special relativity.—Adler, 1987

It appears that the relativistic mass causes interpretation challenges in the context of the Minkowski space-time (four-vector interpretation). Einstein adopted Minkowski[38] space-time by 1922, and in parallel, as it seems, he abandoned the relativistic mass concept. Perhaps, he was happy to do so since Lorentz published the correct relativistic mass already in 1899, which is six years before Einstein published his own theory of relativity. Einstein also got the relativistic mass likely wrong, and other relativity theories, like that of Lorentz, were still considered contenders to the special relativity theory. Still, it seems a bit strange that we cannot divide two sides of an equation by a constant that is already in the formula, namely, to divide the relativistic mass formula of Einstein with c^2, and call the result relativistic mass. Is it forbidden to divide both sides of his formula with a constant that already is there? In a letter to Lincoln Barnett, an American journalist, dated 19 June 1948, Einstein wrote,

It is not good to introduce the concept of the mass $M = m/\sqrt{1-\frac{v^2}{c^2}}$ of a moving body for which no clear definition can be given. It is better to

> *introduce no other mass concept than the "rest mass", m. Instead of introducing M, it is better to mention the expression for the momentum and energy of a body in motion.*

These words have fueled the critics of the relativistic mass concept, see, for example, Ref. 32. However, the arguments against the use of relativistic mass seem rather weak. Perhaps the concept of Minkowski space-time should be subjected to closer scrutiny instead, as it is potentially inconsistent with quantum mechanics.[39] Also, the physical interpretation of the four-momentum and the rest mass "momentum" equated to energy divided by c also requires a careful reassessment, see Ref. 40.

Considering the above, we believe that both the abandonment of the relativistic mass and the setting the special relativity theory in the Minkowski space-time (four vectors) was a mistake that slowed down the progress in physics for many years. We[41,42] have recently shown how a modified relativistic theory can be unified with gravity theory and that quantum mechanics only is consistent with a three-dimensional time (three time-dimensions and three space-dimensions), Inside this framework, relativistic mass leads to no conceptual problems and enables us to work with a more complete mass definition. Our theory is effectively set in a three-dimensional space-time, as the space and time dimensions are just two faces of the same coin. That is, in our model, motion in a given direction happens in both spatial and temporal direction associated with it, e.g., if we move, say, along the x axis in space, we also move along the time axis t_x, but not along t_y or t_z (see the last section before the conclusion).

Adler in 1987 claimed "It should also be pointed out that there is no reason to introduce relativistic mass in general relativity theory" and indeed the relativistic mass was abandoned in today's gravity theory. Also other prominent researchers like Taylor and Wheeler were speaking out against relativistic mass. Many well-known specialists on general relativity theory have not considered how a gravity theory incorporating relativistic mass would perform in terms of predictions and explaining observations. We[43] have recently shown, for example, that by introducing relativistic mass in Newtonian gravity, we can explain supernova data without the need for the dark energy hypothesis. The result arguing the current paradigm naturally cannot be accepted without a careful examination, but it should not be rejected based on prejudice.

Other prominent physicists, such as Rindler,[44,45] who sacrificed most of his career to relativity theory, defended the use of relativistic mass and rebutted the criticism. The famous philosopher of physics, Max Jammer[46] also seemed to approve of its use. Here, we associate with this stance and claim that relativistic mass is an essential part of relativity theory. As it goes hand in hand with relativistic energy, it must be introduced in all fields of physics, not just the gravity theory.

We begin by deriving the relativistic escape velocity from the relativistic modified Newtonian theory, which is given by solving the following equations:

$$\begin{aligned} E_k - G\frac{Mm\gamma}{R} &= 0, \\ mc^2\gamma - mc^2 - G\frac{Mm\gamma}{R} &= 0. \end{aligned} \tag{10}$$

We use Einstein's relativistic kinetic energy, but we also must ensure that the small mass m is relativistic in Newton's gravity formula. That is, we need relativistic mass in gravity theory. Solving with respect to v present in the Lorentz factor γ, we obtain

$$v_e = \sqrt{\frac{2GM}{R} - \frac{G^2M^2}{c^2R^2}}. \tag{11}$$

If we set $v_e = c$ and solve for R we get

$$R = \frac{GM}{c^2}. \tag{12}$$

We can call this result the corrected Schwarzschild radius. However, since we are first[32] to demonstrate this result and have not derived it from the general relativity theory or the Schwarzschild metric, humble as we are, we can call it the Haug radius and use the notation $R_{h,s} = GM/c^2$ (not to be confused with the Hubble radius that we later denote as R_H). It is simply the radius at which the velocity v in the kinetic energy of the mass m offsets the gravitational energy when it is equal to the speed of light, $v = c$. The Schwarzschild radius is the double of this radius, or conversely, the mass has to be twice larger in our model if we keep the radius fixed.

The relativistic *ad hoc* adjustment of the Newtonian formula: $F = GMm\gamma/R^2$ was already suggested in 1981 and 1986 by Bagge[47] and Phipps.[48] However, Peters[49] showed that it predicted only a half of the observed Mercury precession, and thus the idea of using the relativistic modified Newtonian mechanics was abandoned without a deeper exploration. Recently, Corda[50] claimed that he obtained the correct Mercury precession by taking into account the relativistic effects, as well as considering the Mercury and the Sun as a real two-body problem. There are clearly many research results providing arguments in favor of using relativistic masses in the Newtonian framework for its further exploration, see also Ref. 51. For instance, as mentioned above, including relativistic mass effects in the right way can explain supernova data without resorting to the dark energy hypothesis.

It is worth mentioning that for $v_e = c$, replacing R with GM/c^2 (which is its value for the escape velocity v_e), we end up with

$$c = \sqrt{\frac{2GM}{R} - \frac{G^2M^2}{c^2R\left(\frac{GM}{c^2}\right)}} = \sqrt{\frac{GM}{R}}. \tag{13}$$

Our escape velocity formula is identical to the standard orbital speed in a special case when it is equal to c. Therefore, one possible interpretation is that the mass M forms a gravitationally bound system with light, which orbits it at a radius R with velocity c, appearing as a spherical surface of light. No information can likely pass out through such a "fire wall" would be one possible interpretation. As we will see, we perhaps live inside a gigantic sphere with a light shell, known as the Hubble sphere. However, it could be that further relativistic adjustments to the orbital velocity are needed. This area should be investigated further. More likely, as we will show based on these findings, the universe is infinite, but each point at its borders can be reached over time by a distance given by the Hubble radius. In other words, our model gives an information horizon, as we will see.

II. THE MASS DENSITY AND MASS OF THE CRITICAL UNIVERSE

The critical mass density of the observable universe as given by, for example, Weinberg[52] or Ciufolini and Wheeler[53] is

$$\rho_c = \frac{3H_0^2}{8\pi G}, \tag{14}$$

where H_0 is the Hubble constant and ρ_c is the critical mass density. Here, the critical mass density is when the cosmological constant Λ is set as equal to zero as it is for all basic Friedman[54] universes. For refreshment, the Friedmann equation (one of the two) is given by

$$\frac{\dot{a}^2 + kc^2}{a^2} = \frac{8\pi G\rho + \Lambda c^2}{3} \tag{15}$$

by setting the Λ and k both equal to zero and solving with respect to ρ, and we get the well-known critical mass density Eq. (14). The mass in a sphere with this mass density ρ_c is then given by

$$M_c = \rho_c V = \rho_c \frac{4}{3}\pi R_o^3 = \frac{3H_0^2}{8\pi G}\frac{4}{3}\pi R^3 = \frac{c^3}{2GH_0}. \tag{16}$$

Furthermore, if we set the radius of the observable (critical) universe equal to the Hubble radius $R = R_H = c/H_0$, we can rewrite and simplify the equation above as

$$M_c = \frac{3H_0^2}{8\pi G}\frac{4}{3}\pi\left(\frac{c}{H_0}\right)^3 = \frac{1}{2}\frac{c^3}{GH_0}. \tag{17}$$

Another less known way to derive the critical mass density is from the escape velocity by replacing M with a mass density of a sphere. The mass can be described as the mass density multiplied by the volume of the sphere that contains this mass density. That is, we have $M = \rho(4/3)\pi R^3$. Replacing M with this in the general relativity escape velocity formula (or Newton's escape formula), we get

$$v_e = \sqrt{\frac{2G\rho\frac{4}{3}\pi R^3}{R}} = R\sqrt{2G\rho\frac{4}{3}\pi}. \tag{18}$$

Since the Hubble radius is given by $R_H = c/H_0$, if the escape velocity is c and we divide it by R on both sides of the equation and set R equal to the Hubble radius, we get

$$c = R_H\sqrt{2G\rho\frac{4}{3}\pi}, \qquad \frac{c^2}{R_H^2} = 2G\rho\frac{4}{3}\pi. \tag{19}$$

Next, we solve the equation above with respect to ρ and get

$$\rho = \frac{3H_0^2}{8\pi G}, \tag{20}$$

which is the same as the critical mass density we got from the Friedmann equation. Even if this is a less known way to find the critical density, it is described in some sources, see, for example, Ref. 55.

We perform the same operations using our new relativistic escape velocity $v_e = \sqrt{(2GM/R) - G^2M^2/(c^2R^2)}$, and we end up instead with a critical mass density

$$\rho_{h,c} = \frac{3H_0^2}{4\pi G}. \tag{21}$$

As we can see, the critical mass density predicted by our model is twice larger than the critical mass density obtained from the Friedmann equation. The mass of the critical universe based on our full relativistic escape velocity must, therefore, be

$$M_{h,c} = \frac{3H_0^2}{4\pi G}\frac{4}{3}\pi\left(\frac{c}{H_0}\right)^3 = \frac{c^3}{GH_0}. \tag{22}$$

This result is again twice the mass (energy) in the universe than predicted by the Friedman model. Modern cosmological models struggle with the problem of missing baryonic matter[56–58] and, in addition, in order to explain observations, postulate the existence of dark matter, which has not been directly observed. There is also the so-called flatness problem that potentially also could have a new explanation. Our alternative model also could hopefully contribute to explaining these problems. Uncertainty in the Hubble constant also leads to some uncertainty in our mass density, Soltis *et al.*[59] measured the Hubble constant to $H_0 = 72 \pm 2$ (km/s)/Mpc, while, for example, Mukherjee *et al.*[60] in 2020 measured the Hubble constant to be 67.6 ± 4.2 (km/s)/Mpc, so there is a considerably uncertainty in this parameter still, see also Refs. 61–64. For example, the predicted mass with Hubble constant 67 would be approximately 1.85×10^{53} kg. In comparison, a Hubble constant of 72 would mean a mass of approximately 1.72×10^{53} kg, but more importantly, always twice the expected mass as the Friedmann model for the critical universe.

III. DERIVING THE FRIEDMANN EQUATION AND ALSO AN ALTERNATIVE EQUATION FOR THE UNIVERSE FROM ESCAPE VELOCITY

The Friedmann equation for a critical universe is given by (setting Λ and k to zero)

$$\frac{\dot{a}^2}{a^2} = H_0^2 = \frac{8\pi G\rho}{3}. \tag{23}$$

It describes a flat universe. This Friedmann equation was originally derived from the Einstein field equation. Also, the standard escape velocity formula is derived from the general relativity theory. What is not widely known is that the Friedmann equation can be found from the escape velocity formula when setting $v_e = c$, and the radius equal to the Hubble radius $R = R_H$, as follows:

$$\begin{aligned} v_e &= \sqrt{\frac{2GM}{R}}, \\ c &= \sqrt{\frac{2GM}{R_H}}, \\ c &= \sqrt{\frac{8\pi G\rho R_H^2}{3}}, \\ c^2 &= \frac{8\pi G\rho R_H^2}{3}, \\ \frac{c^2}{R_H^2} &= \frac{8\pi G\rho}{3}. \end{aligned} \tag{24}$$

And since $H_0 = c/R_H$, we get

$$H_0^2 = \frac{8\pi G\rho}{3}, \tag{25}$$

which is the well-known Friedmann equation for the flat universe (critical universe). We see this way to derive the Friedmann equation for the critical universe is fully consistent mathematically with the standard way of deriving it from Einstein's field equation. This result is not a big surprise since both the Friedmann equation and the escape velocity formula given above can be derived from the general relativity theory. So also deriving it from the escape velocity can be seen as another way to derive it from Einstein's field equation. However, this way of deriving the Friedmann equation for the critical universe makes it easy to see that the same solution can be derived from standard Newton mechanics. It has the same escape velocity as the general relativity theory. Still, we know that the Newtonian solution involves deriving it from a kinetic energy approximation and nonrelativistic mass assumption that only can be valid for $v \ll c$, namely, $(1/2)mv^2 - G(Mm/R) = 0$. With nonrelativistic mass, we are pointing to that the mass in the Newtonian gravity force formula has no relativistic adjustments, despite m moving at a speed close to c, or even at c when $v = c$ (escape velocity equal to the speed of light). So how can it be that the Friedmann solution gives exactly the same result for a critical universe as a solution from standard nonrelativistic Newton mechanics that we know cannot hold if taking into account relativistic effects when v is close to the speed of light. We think that perhaps the reason is that Einstein ignored relativistic mass (see Section I), something that naturally should be studied more carefully before any final conclusion is made.

Einstein's field equation for a weak gravitational field gives the same results as Newton's equations. The Hubble sphere or, in more general terms, supermassive black holes are a very interesting case. The Hubble sphere has mathematical properties of a black hole. The gravitational acceleration for supermassive black holes and the Hubble sphere is extremely weak at the Hubble radius, which is equal to the Schwarzschild radius. For example, if the Hubble constant is 70, then the gravitational acceleration at the Hubble radius is only 3.40×10^{-10} m/s^2 in general relativity theory and twice of that in our theory. This result is very small compared with gravitational acceleration, for example, at the Earth's surface, which is about 9.8 m/s^2, which is a weak gravitational field. The escape velocity at the Earth's surface is insignificant compared with the speed of light; however, the escape velocity at the Hubble radius is c in the critical universe. The Hubble sphere and any super large Schwarzschild sphere have properties of a weak gravitational field (the gravitational acceleration) and at the same time properties where relativistic effects should be of great importance, namely, the escape velocity.

An interesting question is what type of an equation similar to Friedmann we will get when we derive it from the relativistic escape velocity given in Section I, Eq. (11), by simply replacing R with the Hubble radius that must be identical to the radius at which the escape velocity is c. We indeed obtain

$$\begin{aligned} v_e &= \sqrt{\frac{2GM}{R} - \frac{G^2M^2}{c^2R^2}}, \\ c &= \sqrt{\frac{2GM_c}{R_H} - \frac{G^2M_c^2}{c^2R_H^2}}, \\ c &= \sqrt{\frac{8\pi G\rho R_H^2}{3} - \frac{16\pi^2G^2\rho^2R_H^6}{9c^2R_H^2}}, \\ c^2 &= \frac{8\pi G\rho R_H^2}{3} - \frac{16\pi^2G^2\rho^2R_H^4}{9c^2}, \\ \frac{c^2}{R_H^2} &= \frac{8\pi G\rho}{3} - \frac{16\pi^2G^2\rho^2R_H^2}{9c^2}, \\ H_0^2 &= \frac{8\pi G\rho}{3} - \frac{16\pi^2G^2\rho^2}{9H_0^2}. \end{aligned} \tag{26}$$

This can be simplified further to

$$H_0^2 = \frac{4\pi G\rho}{3}. \tag{27}$$

The last simplification is perhaps easiest seen by study Section II [Eq. (13)], where we know that when $v_e = c$, we can simplify our escape velocity formula to

$$v_e = c = \sqrt{\frac{2GM}{R} - \frac{G^2M^2}{c^2R^2}} = \sqrt{\frac{GM}{R}},$$

and therefore, we have

$$\begin{aligned} c &= \sqrt{\frac{GM_{c,h}}{R_H}}, \\ c &= \sqrt{\frac{4\pi G\rho R_H^2}{3}}, \\ \frac{c^2}{R_H} &= \frac{4\pi G\rho}{3}, \\ H_0^2 &= \frac{4\pi G\rho}{3}. \end{aligned} \tag{28}$$

This result can be seen as an alternative to the Friedman critical universe equation (and, therefore, even to the general relativity theory). The equation above is not from Newton's theory, which would give the same escape velocity as general relativity and the same universe solution as the Friedmann solution for a critical universe. The equation above is from relativistic adjusted Newtonian theory, where we take into account relativistic kinetic energy and relativistic mass. Our result should be carefully investigated at this stage, for example, the implications that it predicts. As we can measure the Hubble constant, we can solve for ρ. This procedure gives

$$\rho_{h,c} = \rho = \frac{3H_0^2}{4\pi G}. \tag{29}$$

This result is naturally the same result we got in Eq. (21). Again, we highlight that this result predicts that the mass and mass density inside the Hubble sphere without any inflation is twice that of what is predicted by the Friedmann equation. Both equations have the same Hubble radius, and in both models, the Hubble radius is the radius where the escape velocity is c. But our new theory gives an escape velocity of c at a radius equal to $R_{h,s} = GM/c^2$ while general relativity predicts that this is at the Schwarzschild radius $R_s = 2GM/c^2$. This result can only be true if the mass density in our model is twice that in the Friedmann solution, which is the case as we have demonstrated to be mathematically consistent with our theory.

From our new universe equation, we can understand that the so-called scaling parameter in the Friedmann equation, in reality likely represents that the redshift is a function of how far we are from the Hubble sphere radius. The Hubble sphere radius is then likely an information horizon, beyond which we cannot get any information. That is about all. We think that the interpretation that it means the universe is expanding could be incorrect.

IV. MORE FORMAL DERIVATION OF THE FRIEDEMANN EQUATION FROM NEWTON MECHANICS AND THE HAUG EQUATION FROM RELATIVISTIC MODIFIED NEWTON THEORY

It is well known the Friedmann equation can also be derived from Newton mechanics, which is likely because the general relativity theory is considered to be Newtonian in the weak field limit. The Friedmann equation can be derived from Newtonian mechanics as

$$\begin{aligned} U = T + V &= \frac{1}{2}m\dot{R}^2 - \frac{GMm}{R}, \\ &= \frac{1}{2}m\dot{R}^2 - \frac{4\pi}{3}G\rho R^2 m. \end{aligned} \tag{30}$$

Assume $\overrightarrow{R}(t) = a(t)\overrightarrow{x}$ and substitute this in the equation above, and we get

$$U = \frac{1}{2}m\dot{a}^2x^2 - \frac{4\pi}{3}G\rho a^2x^2m. \tag{31}$$

We can rearrange this, and we get

$$\begin{aligned} \frac{\dot{a}^2}{a^2} &= \frac{8\pi}{3}G\rho - \frac{kc^2}{a^2}, \\ H_0^2 &= \frac{8\pi}{3}G\rho - \frac{kc^2}{a^2}, \end{aligned} \tag{32}$$

where $kc^2 = -2U/(x^2m)$. The equation above is the Friedmann equation including the constant k. When setting $k=0$ we get the critical universe solution that we also got from the escape velocity formula.

If we take into account relativistic energy as well as relativistic mass, we get

$$\begin{aligned} U &= T + V = m\dot{R}^2\gamma - m\dot{R}^2 - \frac{GMm\gamma}{R}, \\ U &= m\dot{R}^2\gamma - m\dot{R}^2 - \frac{4\pi}{3}G\rho R^2 m\gamma, \\ U &= m\gamma\dot{a}^2x^2 - m\dot{a}^2x^2 - \frac{4\pi}{3}G\rho a^2x^2m\gamma, \\ \gamma\frac{\dot{a}^2}{a^2} &- \frac{\dot{a}^2}{a^2} = \frac{4\pi}{3}G\rho\gamma - \frac{kc^2}{a^2}, \end{aligned} \tag{33}$$

where $\gamma = 1/\sqrt{1 - v^2/c^2}$ and v is the velocity of m, and where $kc^2 = -U/(x^2m)$. Divide by γ on both sides and we get

$$\frac{\dot{a}^2}{a^2} - \frac{\dot{a}^2}{a^2}\sqrt{1 - \frac{v^2}{c^2}} = \frac{4\pi}{3}G\rho - \frac{kc^2}{a^2}\sqrt{1 - \frac{v^2}{c^2}}. \tag{34}$$

For $v_e = v = c$, that is when we are at the Hubble radius, we obtain

$$\begin{aligned} \frac{\dot{a}^2}{a^2} &= \frac{4\pi G\rho}{3}, \\ H_0^2 &= \frac{4\pi G\rho}{3}. \end{aligned} \tag{35}$$

This is the same solution as we got from the escape velocity formula. Where as in the Friedmann solution it is an open question what k should be set to, in our full relativistic framework, the solution is independent of k. This means that this constant has no impact on our universe when our theory is linked to the Hubble scale. Consequently, the universe in a full relativistic Newtonian mechanics can likely not be expanding forever. Even the Big Bang theory can then be questioned.

V. ANY MASS DENSITY ABOVE ZERO IN A LARGE AREA OF SPACE ALWAYS HAS A SCHWARZSCHILD RADIUS (SPHERE)

That our observable universe is the interior of a gigantic black hole has been suggested already in 1972 by Pathria,[65] who wrote "the universe may not only be a closed structure (as perceived by its inhabitants at the present epoch) but may also be a black hole, confined to a localized region of space which cannot expand without limit." Several other researchers also considered this idea,[66–68] for example, Kip Thorn said we have enough mass inside a sphere somewhat larger than 10 billion light-years out, for this to be the case, and therefore we could, theoretically, live inside a black hole or what he calls a reverse black hole, a so-called white hole, but he considers this rather improbable.[69] We think that the idea that we live inside the sphere with mathematical properties identical or similar to a black hole should not be excluded. However, the existent theories of the interiors of such "structures" are very simplistic, if not speculative, and usually set in the general relativity theory, despite other frameworks providing similar solutions of spheres with escape velocity c, as we have shown in Section IV. In our theory, we get different results, which lead us a very different interpretation of how such structures are linked to the observable universe and the Planck scale.

Assume a very large area of the universe or even an infinite universe with a given average density. The mass density in the surface or center of the Earth is clearly much higher than the mass density, for example, at the mid-point between Earth and the Moon. However, inside an enormous space volume covering millions of galaxies, we can calculate an average density that is basically the same if we split that large volume in two or even ten. This is the cosmological principle that is empirically justified on scales larger than 100 Mpc. For a given universe mass density, even if it is very small, the mass will increase as a function of the volume at which we look. If we look at the volume inside a spherical shape, the mass for a given density will increase by R^3 as we increase the radius, namely, proportionately the volume of a sphere is $V = 4/3\pi R^3$.

On the other hand, the Schwarzschild radius R_s is a linear function of M, which means any large space with a given mass density must have a Schwarzschild radius (and a Haug radius), something we will now investigate in detail. One can argue that since gravity bends space, we must go beyond spherical shapes to discuss this problem. However, in a critical universe, even based on the general relativity theory, i.e., the Friedmann solution for the critical universe, we are still operating in Euclidean geometry.

A mass M uniformly filling a sphere of a radius R with density ρ is

$$M = \rho \frac{4}{3}\pi R^3. \tag{36}$$

Thus, the escape velocity from a sphere filled with the mass of this density is given by

$$\begin{aligned} v_e &= \sqrt{\frac{2GM}{R}}, \\ v_e &= \sqrt{\frac{2G\rho\frac{4}{3}\pi R^3}{R}}. \end{aligned} \tag{37}$$

The escape velocity of c is the maximum escape velocity. If we set $v_e = c$, keeping the mass density ρ constant and solving this with respect to R to obtain what must be the Schwarzschild radius, as follows:

$$\begin{aligned} v_e &= c = \sqrt{\frac{2G\rho\frac{4}{3}\pi R_s^3}{R_s}}, \\ R_s^2 &= \frac{c^2}{2G\rho\frac{4}{3}\pi}, \\ R_s &= \sqrt{\frac{c^2}{2G\rho\frac{4}{3}\pi}}. \end{aligned} \tag{38}$$

If we input ρ equal to the critical mass density of the observable universe, we get

$$R_s = \sqrt{\frac{c^2}{2G\frac{3H_0^2}{8\pi G}\frac{4}{3}\pi}} = \sqrt{\frac{c^2}{H_0^2}} = \frac{c}{H_0}. \tag{39}$$

This equation is the Schwarzschild radius of a universe with a mass density equal to the critical mass density, which is equal to the Hubble radius. The Haug radius based on relativistic mass adjustments is $R_{h,s} = GM/c^2$, but this will also be identical to the Hubble radius, as the mass density in our model is twice that of the standard model

$$\begin{aligned} v_e &= c = \sqrt{\frac{2GM_{h,c}}{R} - \frac{G^2M_{h,c}^2}{c^4R^2}}, \\ c &= \sqrt{\frac{2G\frac{c^3}{GH_0}}{R} - \frac{G^2\left(\frac{c^3}{GH_0}\right)^2}{c^2R^2}}, \\ c &= \sqrt{\frac{2c^3}{H_0R} - \frac{c^4}{R^2H_0^2}}. \end{aligned} \tag{40}$$

This gives $R = R_{h,s} = GM_{h,c}/c^2 = c/H_0$, which is identical to the Hubble radius for the Hubble sphere. We also have $R_s = 2GM_c/c^2 = c/H_0$ from general relativity theory that is consistent with the Friedmann solution and the Schwarzschild solution. Since H_0 is indirectly observed empirically, this point can only be true if the predicted mass inside the Hubble sphere in our new model is twice as large as in the Friedmann model. This result also explains why the Hubble radius is so special. One possible interpretation is that every point in an infinite universe can only be reached by light (information) that comes at maximum the Hubble radius distance. Each point in space then has an information horizon.

VI. HALF THE SCHWARZSCHILD TIME IS THE MAXIMUM ACCELERATION TIME FOR THE SCHWARZSCHILD RADIUS ACCELERATION

What we can call the Schwarzschild time is simply the Schwarzschild radius divided by the speed of light. In other words, it is time that it takes for light to travel the distance equal to the Schwarzschild radius

$$T_s = \frac{R_s}{c} = \frac{2GM}{c^3}. \tag{41}$$

Interestingly, the Hubble constant is also one divided by the Schwarzschild time, that is,

$$H_0 = \frac{1}{T_s} = \frac{1}{\frac{2GM_u}{c^3}} = \frac{c}{R_s} = \frac{c}{R_H}. \tag{42}$$

The Hubble constant can also be interpreted as a frequency. At the Schwarzschild radius, the gravitational acceleration field strength is given by

$$g = \frac{GM_c}{R_s^2} = \frac{GM_c}{R_H^2}. \tag{43}$$

We can call this phenomenon the Schwarzschild acceleration. Interestingly, it takes twice the Schwarzschild time, which is identical to twice the Hubble time to accelerate a particle from zero to c at this acceleration rate. That is, we have

$$\begin{aligned} c &= 2T_s a_s = \frac{2}{H_0} a_s = 2T_s \frac{GM}{R_s^2} \\ &= 2\frac{2GM}{c^3}\frac{GM}{4G^2M^2/c^4}. \end{aligned} \tag{44}$$

Since it takes twice the Hubble time to accelerate to the speed of light, one may easily arrive at an incorrect conclusion that space must expand. When using the Haug radius that is calculated from the full-relativistic escape velocity, the Haug acceleration is given by

$$g = \frac{GM_{h,c}}{R_{h,s}^2} = \frac{GM_{h,c}}{R_H^2}. \tag{45}$$

It takes the Haug time $T_{h,s} = R_{h,s}/c = GM/c^3 = R_H/c$, at the Haug acceleration to accelerate a particle from zero to c, that is, we have

$$\begin{aligned} T_{h,s}a_{h,s} &= \frac{1}{H_0} a_{h,s} = T_{h,s}\frac{GM_{h,c}}{R_{h,s}^2} \\ &= \frac{GM_{h,c}}{c^3}\frac{GM_{h,c}}{G^2M_{h,c}^2/c^4} = c \end{aligned}. \tag{46}$$

This procedure means our new model predicts that a mass located at or very close to the corrected Hubble radius leaving the Hubble radius will be reaching speed c in the Hubble time (Hubble age of the universe). This result perhaps indicates that each point in an infinite universe can be affected only by information at the Hubble distance apart. We underline that in our new model it takes the Hubble time to accelerate from 0 to c, while in the standard model, it takes twice the Hubble time. In our view, the standard model is likely incomplete. That a particle cannot reach c at this acceleration at the Hubble time can be misinterpreted as the universe is expanding. Our new model does not seem to need any expansion of space and still be internally consistent. However, these physical conclusions require further investigation, and we encourage other researchers to explore them.

The cosmological redshift has been interpreted as the space expansion at the following velocity (the Hubble flow where $D \ll R_H$):

$$v_H = H_0 D. \tag{47}$$

This again is linked to cosmological red-shift by

$$z \approx \frac{v}{c}. \tag{48}$$

However, we can also rewrite this as

$$z \approx \frac{v}{c} = \frac{D\bar{\lambda}_H}{l_p^2} = \frac{Dc^2}{GM_{h,c}} = \frac{1}{\frac{GM_{h,c}}{Dc^2}}, \tag{49}$$

where $M_{h,c}$ is the Haug critical mass and $\bar{\lambda}_H$is the reduced Compton wavelength of the Haug critical mass. Hence, one possibility is that the observed cosmological redshift is not related to expanding space. Some will possibly recognize the denominator as the formula for gravitational redshift, but this will be a type of inverse gravitational redshift as observed from the inside of the Hubble sphere.

VII. SIMILARITY UNSOLVED CHALLENGES BETWEEN THE HUBBLE SCALE AND THE PLANCK SCALE IN GENERAL RELATIVITY THEORY

When Max Planck[70,71] introduced the Planck mass (and the Planck length and Planck time) in 1899 and considered it fundamental to natural sciences, he was not very clear on what it could represent in reality. Could it, for example, be linked to a particle? Loyd Motz[72,73] while working at Rutherford laboratory, was possibly the first to suggest that the Planck mass could be linked to a particle, the Planck mass particle or Uniton, as he coined it. Motz naturally understood that the Planck mass, approximately 2.17×10^{-8} kg, was enormous in mass compared with any observed particle, so he suggested that such a particle existed only right after the Big Bang and then immediately radiated most of its energy into today's observed particles, such as the electron and the proton. Some years later, Hawking[74] in 1971 and Motz and Epstein[75] in 1979 suggested instead that the Planck mass could be a micro black hole, something we could possibly call a Planck mass particle. However, a Soviet scientist Moisey Markov,[76] unknown to many researchers in the West, described the Planck mass-size micro black holes several years before Hawking, already in 1966/67, and we would say in a more rigorous and complete way. Again, a black hole is defined as a mass enclosed inside a small enough radius to make the escape velocity at this radius be exactly c. The mass that gives an escape velocity radius equal to the Planck length is half the Planck mass in general relativity theory (at least in the Schwarzschild solution). In our new theory, the Planck mass gives an escape velocity radius (where $v_e = c$) equal to the Planck length. In the general relativity theory, the Planck mass does not seem that unique. The Planck mass particle has a reduced Compton wavelength equal to the

Planck length $\bar{\lambda} = \hbar/(m_p c) = l_p$, but the escape velocity at this radius for this mass is above c in general relativity theory, namely, $v_e = \sqrt{2Gm_p/l_p} = c\sqrt{2}$. This point seems to make it impossible for the Planck mass particle to exist under general relativity theory or get as close to it as the Planck length or the reduced Compton wavelength of the particle. Hawking likely purposely indicated that the micro black hole is only approximately equal to the Planck mass. He could possibly see that general relativity theory did not perfectly match the Planck mass's mathematical properties. Motz and Eppstein suggested the micro black hole to have half the Planck mass. This is likely because half the Planck mass has the properties of a black hole at a distance equal to the Planck length in general relativity theory, and not the full Planck mass.

Papers discussing micro black holes are often diffused on the exact mass of this object. They say, that it is close to the Planck mass, but clearly, it seems that the standard theory does not make the Planck mass unique. In our theory, the escape velocity at the Planck length for the Planck mass is exactly c. That is, we have

$$v_e = \sqrt{\frac{Gm_p}{l_p} - \frac{G^2 m_p^2}{c^2 l_p^2}} = c. \tag{50}$$

We find the Hubble sphere's mass density in the standard theory to be the half of what we find when using our new relativistic escape velocity formula. We can also find the Hubble constant equivalent of a spherical Planck mass

TABLE I. This table shows a summary of General relativity in relation to the Planck scale and the Hubble scale.

Escape velocity	$v_e = \sqrt{\frac{2GM}{R}}$	$v_e = \sqrt{\frac{2GM}{R}}$
Radius where $v_e = c$	$R_s = \frac{2GM}{c^2}$	$R_s = \frac{2GM}{c^2}$
	Planck mass	**Half Planck mass**
Mass	$m_p = \sqrt{\frac{\hbar c}{G}}$	$\frac{1}{2}m_p = \frac{1}{2}\sqrt{\frac{\hbar c}{G}}$
Schwarzschild radius	$R_s = 2l_p$	$R_s = l_p$
Gravitational acceleration in Planck time	$g_s t_p = \frac{1}{4}c$	$g_s t_p = \frac{1}{2}c$
Gravitational acceleration in Schwarzschild time	$g_s t_s = \frac{1}{2}c$	$g_s t_s = \frac{1}{2}c$
Escape velocity at l_p	$v_e = c\sqrt{2}$	$v_e = c$
Planck scale equation	$\frac{1}{t_p^2} = H_p^2 = \frac{32\pi G\rho_{p,s}}{3}$	$\frac{1}{t_p^2} = H_p^2 = \frac{8\pi G\rho_{p,s}}{3}$
	$\rho_{p,s} = \frac{m_p}{\frac{4}{3}\pi(2l_p)^3}$	$\rho_{p,s} = \frac{\frac{1}{2}m_p}{\frac{4}{3}\pi l_p^3}$
Conclusion	Do not match Planck scale	Do not match Planck scale
Possible (but "absurd") solution	Expanding space?	Expanding space
Solution 2	GR incomplete	GR Incomplete
	Universe mass	**Universe mass**
	Alternative theory:	Standard theory:
Mass	$M_c = \frac{c^3}{GH_0}$	$M_c = \frac{1}{2}\frac{c^3}{GH_0}$
Radius where $v_e = c$	$R_{h,s} = \frac{c}{H_0} = R_H$	$R_s = \frac{c}{H_0} = R_H$
Gravitational acceleration in Hubble time at R_H	$g_H t_s = c$	$g_H t_s = \frac{1}{2}c$
Gravitational acceleration in Hubble time at R_s	$g_s t_H = \frac{1}{4}c$	$g_s t_H = \frac{1}{2}c$
Gravitational acceleration in Schwarzschild time atR_s	$g_s t_s = \frac{1}{2}c$	$g_s t_s = \frac{1}{2}c$
Escape velocity at R_s	$v_e = c\sqrt{2}$	$v_e = c$
Universe equation	$\frac{1}{\frac{R_H^2}{c^2}} = \frac{1}{t_H^2} = H_0^2 = \frac{32\pi G\rho}{3}$	$\frac{1}{t_H^2} = H_0^2 = \frac{8\pi G\rho}{3}$
		Friedmann critical universe
Conclusion	Do not match up	Do not match up
Solution 1	Expanding space?	Expanding space?
Solution 2	GR incomplete	GR incomplete

particle. In a similar manner to how we derived our Hubble sphere equation, we set $R=l_p$ and $v_e=c$, first in our new escape velocity formula and get

$$H_{0,p}=\frac{4\pi c^3}{3G\rho_p}=\frac{1}{t_p}=\frac{c}{l_p}. \tag{51}$$

Thus, the Hubble constant equivalent for the Planck particle is the Planck frequency. Based on the standard escape velocity from the general relativity theory, one gets

$$H_{0,p}=\frac{8\pi c^3}{3G\rho_p}=\frac{1}{\frac{1}{2}t_p}=\frac{2c}{l_p}. \tag{52}$$

Twice the Planck frequency is again likely impossible as the Planck length is the minimum length and the maximum speed of light is c in the Universe, and hence the maximum frequency should be $f=c/l_p$ (the Planck frequency). Alternatively, one must introduce the expansion of space to make the formula make sense. A third alternative is not to have the Planck mass density but half the Planck mass density inside the Planck sphere. General relativity, in our view, gives strange predictions when approaching the Planck scale. This concern is, in our view, much of the same issues as one has with the Hubble sphere in general relativity theory. However, it is more easily seen theoretically at the Planck scale because one can see that the Planck mass cannot be a unique object in general relativity theory. It requires alterations to fit the formula: either one must resign from the Planck length uniqueness, or one must release the maximum speed limit c (and introduce expansion), or one must reduce the unique mass to half the Planck mass. Our new theory perfectly lines up at the same time with the Planck mass, the Planck length, and the Planck time. Our theory seems to apply to the whole range, from the Planck scale to the Hubble sphere. Table I summarizes similarities between the Planck scale issues and Hubble scale issues in general relativity theory, as we can see general relativity do not seem to fit the Planck scale well, nor actually the Hubble scale that well.

Table II summarize our new theory where we take into account relativistic mass and a full relativistic escape velocity. Our new theory gives a perfect match to the Planck scale, and also the match to the Hubble scale is more elegant than under general relativity theory.

TABLE II. This table shows a summary of full relativistic escape velocity in relation to the Planck scale and the Hubble scale.

Escape velocity	$v_e=\sqrt{\frac{2GM}{R}-\frac{G^2M^2}{c^2R^2}}$
Radius where $v_e=c$	$R_{h,s}=\frac{GM}{c^2}$
Haug time	$t_{h,s}=\frac{R_{h,s}}{c}$
	Planck mass
Mass	$m_p=\sqrt{\frac{\hbar c}{G}}$
Haug radius	$R_{h,s}=\frac{Gm_p}{c^2}=l_p$
Gravitational acceleration in Planck time at $R_{h,s}$	$g_{h,s}t_p=c$
Gravitational acceleration in Haug time	$g_{h,s}t_{h,s}=c$
Escape velocity at l_p	$v_e=c$
Planck scale equation	$\frac{1}{t_p^2}=H_p^2=\frac{4\pi G\rho_p}{3}$
Planck mass density inside $R_{h,s}$	$\rho_p=\frac{m_p}{\frac{4}{3}\pi l_p^3}$
Conclusion	Match Planck scale
	Universe mass
Mass	$M_{h,c}=\frac{c^3}{GH_0}$
Haug radius	$R_{h,s}=R_H$
Gravitational acceleration in Hubble time at R_H	$g_Ht_H=\frac{GM_h}{R_H^2}\frac{R_H}{c}=c$
Gravitational acceleration in Haug time at $R_{h,s}$	$g_{h,s}t_{h,s}=\frac{GM_h}{R_{h,s}^2}\frac{R_{h,s}}{c}=c$
Escape velocity as $R_{h,s}$	$v_e=c$
Universe equation	$H_0^2=\frac{4\pi G\rho}{3}$
	Haug universe
Conclusion	Match Hubble scale

VIII. CONCLUSION

The general relativity and the Newtonian mechanics lead to the same escape velocity formula. However, these derivations seem to ignore the concept of the relativistic mass. Since the introduction of the relativistic mass by Lorentz, there has been a long discussion of whether it should be used. Einstein was negative about it and seemed to avoid it in developing his gravity theory. We show that introducing the relativistic mass to the Newtonian framework, leads us to a different escape velocity than the general relativity theory. We call our new escape velocity the full relativistic escape velocity, as it also applies to the relativistic mass. We have shown how one can derive the Friedmann equation for the critical universe from the standard escape velocity, and we derive a similar equation from our full relativistic escape velocity formula. Our formula predicts that the density of the critical universe mass (energy) is twice larger than predicted by the standard model. Just as importantly, we demonstrated that the curvature constant k (found in the Friedmann equation) disappears from the Universe equations when one takes into account relativistic mass. In addition, we would like to emphasize that using the relativistic mass, allows us to almost perfectly explain supernova data without resorting to dark energy. Our model also predicts that the Planck mass particle has an escape velocity of c at the Planck length, in contrast to the general relativity theory, which leads to the escape velocity exceeding the speed of light (can "only" be explained away by introducing the hypothesis of expanding space). Therefore, the so-called micro black holes are different from the Planck mass, or must have a radius of twice the Planck length, which would require changes to Planck units. In short, the general relativity description of micro black holes does not comply with all Planck units, while our theory does. Consequently, our model leads to a different interpretation of cosmology than the standard model. In particular, our new Universe equation seems to be consistent, without the need for cosmic expansion or even a Big Bang event. Although further work is needed to establish our approach, we think that our findings are already interesting enough to be presented to the scientific community for discussion and feedback.

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