Why the Sagnac effect favors absolute over relative simultaneity

Gianfranco Spavieri[1,a)] and Espen Gaarder Haug[2,b)]
[1]*Centro de Física Fundamental, Universidad de Los Andes, Mérida 5101, Venezuela*
[2]*Norwegian University of Life Sciences, Chr. Magnus Falsens vei 18, 1430 Ås, Norway*

(Received 21 May 2019; accepted 25 July 2019; published online 29 August 2019)

Abstract: We consider a thought experiment, equivalent to the Sagnac effect, where a light signal performs a round trip over a closed path. If special relativity (SR) adopts Einstein synchronization, the result of the experiment shows that the local light speed cannot be c in every section of the closed path. No inconsistencies are found when adopting absolute synchronization. Since Einstein and absolute synchronizations can be discriminated, the conventionality of the one-way speed of light holds no longer. Thus, as sustained by specialists, it might be a viable formulation of SR that reinstates the conservation of simultaneity, even though it allows for relativistic effects, such as time dilation. Such an approach may lead to the discovery of new effects and a better understanding of relativistic theories. © *2019 Physics Essays Publication*.
[http://dx.doi.org/10.4006/0836-1398-32.3.331]

Résumé: Nous considérons une expérience de pensée, équivalente à l'effet Sagnac, dans laquelle un signal lumineux effectue un aller-retour sur un chemin fermé. Si la relativité restreinte (RR) adopte la synchronisation d'Einstein, le résultat de l'expérience montre que la vitesse de la lumière locale ne peut pas être c dans chaque section du chemin fermé. Aucune incohérence n'est constatée lors de l'adoption de la synchronisation absolue. Comme Einstein et les synchronisations absolues peuvent être discriminées, la conventionalité de la vitesse de la lumière dans un sens ne tient plus. Ainsi, comme le soutiennent les spécialistes, il pourrait s'agir d'une formulation viable de la RR qui rétablisse la conservation de la simultanéité, même si elle permet des effets relativistes, tels que la dilatation du temps. Une telle approche peut conduire à la découverte de nouveaux effets et à une meilleure compréhension des théories relativistes.

Key words: Relativistic Theories; One-Way Speed of Light; Einstein Synchronization; Sagnac Effect.

I. INTRODUCTION

The idea of formulating special relativity (SR) by means of coordinate transformations that reflect absolute simultaneity is not new. It has been proposed by Mansouri and Sexl[1]in the context of the conventionalist thesis, where the one-way speed of light is conventional. More recently, Kassner[2] has suggested to interpret the Sagnac effect[3] by adopting absolute clock synchronization for SR. The difficulties involved in the measurement of the one-way speed of light and the conventionality of clock synchronization in SR have been discussed previously by Poincaré,[4] Einstein,[5] Reichenbach,[6] and Grünbaum.[7] Later on, Mansouri and Sexl considered relativistic theories based on transformations that depend on an arbitrary synchronization parameter, although maintaining the same rod-contraction and clock-retardation as the Lorentz transformation (LT). Due to the conventionality of the one-way speed c, "preferred frame" relativistic theories based on absolute (or "external") synchronization are capable of interpreting all the experiments supporting *standard* SR, i.e., SR with ("internal") Einstein synchronization. Therefore, according to the conventionalist thesis, since relativistic transformations adopting absolute synchronization[8] provide the same observable results as the LT,[9–14] SR with absolute synchronization is physically equivalent to SR with Einstein synchronization: All of the experiments supporting standard SR also support SR with absolute synchronization.

Further problems with Einstein synchronization were pointed out by Sagnac,[3] who claimed that his experiment—where light performs a closed path on the circumference of a rotating disk [Fig. 1(a)]—disproves standard SR. Later, Sagnac's contentions were highlighted by Selleri's[15] paradox, which indicates that the interpretation of the Sagnac Effect requires the local one-way speed of light to be $c+v$ or $c-v$ at a point of the disk circumference moving with velocity $v=\omega R$. Similar inconsistencies, concerning the failure of Einstein synchronization when performed along a closed path, have been pointed out by Kassner,[2] Gift,[16] and earlier observed by Weber[17] and by Anandan.[18] Thus, according to the experimental observation and theoretical considerations of many authors, there are valid reasons to question the validity of Einstein's second postulate about the constancy of the speed of light c. As is well-known, a paradigm change, if appropriate, has to overcome the inertia involved in accepting new postulates and abandoning old ones. If, as shown below, the mentioned inconsistencies are present, as a fundamental theory SR remains valid, but the clock synchronization to be adopted with SR needs to be changed. In effect, the presence of inconsistencies is sufficient to call for a change of the time paradigm in SR, which consists of replacing Einstein synchronization and relative

a)gspavieri@gmail.com
b)espenhaug@mac.com

simultaneity with absolute synchronization and simultaneity. So far, this paradigm change has been accepted by a few specialists only, due to the inertia of mainstream journals and the physics community toward accepting the change. Additional considerations include the fact that the issue is not yet well known and, quite importantly, the adherence to the aforementioned conventionalist belief that Einstein synchronization and absolute synchronization are physically equivalent.

The outcome of the Sagnac effect is foreseen by SR with either Einstein or absolute synchronization, because the two synchronizations provide the same result for light signals moving on a closed path. Interestingly, in order to solve Selleri's paradox,[15] Kassner[2] suggests that SR adopt absolute synchronization in interpreting the Sagnac effect. In this way, Kassner dismisses Selleri's paradox on the basis that the Sagnac effect can be equivalently described with either Einstein or absolute synchronization. Nevertheless, a turning point concerning clock synchronization has been achieved recently by Spavieri *et al.*,[19] who show that, in general, Einstein and absolute synchronization predict different observable results. Therefore, it turns out that the two synchronizations are not physically equivalent: Only one of the two is correct!

Among other sources, a coherent and exhaustive description of the Sagnac Effect [Fig. 1(a)], including its linear version, has been given by Field.[20] In light of the achievement of Ref. 19, in the present work, we analyze the Sagnac Effect in its linear form [Fig. 1(b)] in order to check which synchronization describes it coherently and without signs of internal inconsistencies. One of our aims is to make the issue of synchronization better known and accessible to a broader audience. Furthermore, we show below that, in describing the behavior of the light signal, the consequences of nonconservation of simultaneity and the inconsistencies related to Einstein synchronization are clear-cut and easily understood. No inconsistencies are found with absolute simultaneity.

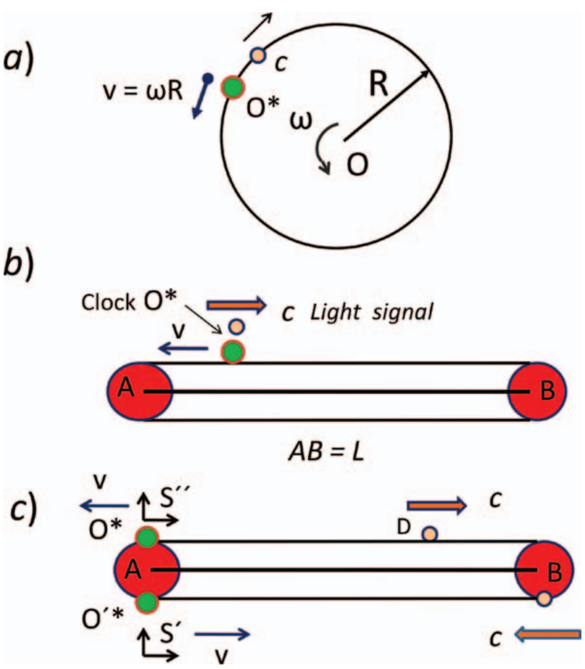

FIG. 1. (a) In the Sagnac effect, a counter-propagating light signal is emitted by clock O^* co-moving with a rotating disk. (b) In the linear Sagnac Effect, clock O^* is co-moving with the conveyor belt, sliding with velocity v on the pulleys of the arm AB, while a counter-propagating light signal is emitted from O^*. (c) When O^* is on the upper part of the belt, is co-moving with frame S'' in relative motion with velocity $-v$ with respect to AB, and co-moving with S' when on the lower part. According to frame S', the light signal emitted by O^* reaches point B at the same instant of time when point A reaches clock $O^* \equiv O'^*$, which displays the time $t''^* = t'^* = 0$ and the light signal return trip begins. According to frame S'', instead, when the clocks coincide at point A at the time $t''^* = t'^* = 0$, due to nonconservation of simultaneity the signal is at point D.

Beyond discussions that involve paradoxes or inconsistencies, the idea of adopting absolute simultaneity for SR is a feasible alternative for SR.[1,2] In effect, as done in Ref. 19, if the results of models developed with the LT are contrasted with the results obtained with absolute simultaneity, meaningful observable differences might emerge. This approach to SR may then lead to the discovery of new effects and to a more comprehensive vision of relativistic theories and natural phenomena.

Before moving on to Sec. II, where we use the expression "time t," as done by the majority of physicists, after the suggestion of one of the Referees we point out that it was never used by Einstein, who rather used his own "clock variable," as has been clearly and lucidly pointed out recently by Lundberg.[21] Since we have been talking about clock synchronization, by using the term clock variable, instead of time t, it becomes easier to understand the difference between "coordinating clocks" and "synchronizing clocks." In the words of Lundberg, Einstein synchronization may be criticized as follows:

> "I have said several times that Einstein specified a procedure for coordinating clocks, but I have not said that he specified a procedure for synchronizing clocks. 'Coordinating' is a safe and unproblematic word in this context. To coordinate clocks is merely to connect the clock settings in some way or other, so that they are not independent of each other. Einstein's procedure clearly does that. 'Synchronizing' is more specific than 'coordinating,' but in a way that is not obvious."

II. PROPAGATION OF A LIGHT SIGNAL IN THE LINEAR SAGNAC EFFECT

Let us consider the conveyor belt of Fig. 1(b), where the clock O* is fixed to the upper section of the belt, moving with velocity $-v$ relatively to the frame S where the conveyor arm AB of length L is stationary. A light signal is emitted by an observer co-moving with O* toward point B. When O*, co-moving with the rotating belt, reaches point A, it changes direction and is now in the lower section of the belt, moving toward point B. The conveyor belt system can be shown to be equivalent to the "linear" Sagnac effect that can be approached in a different way.[22] In the present approach, we denote by S'' the inertial frame of which $O^* \equiv O''^* \equiv O''$ is the origin, as long as O* stays on the upper section of the belt, while the inertial frame of $O^* \equiv O'^* \equiv O'$ in the lower section of the belt is denoted by S'. The time measured by clock O^* is denoted by t^*, while the time measured by clocks of S'' is denoted by t'' and t' corresponds to the time measured by clocks of S'. Thus, when O^* is on the upper section of the belt and is co-moving with S'', we have $t''^* = t''$, while $t'^* = t'$ when O^* is on the lower section of the belt and is co-moving with S'. The two clocks at the origin of S'' and S', $O''^* \equiv O''$ and $O'^* \equiv O'$, respectively, coincide when their readings are set at $t''^* = t'' = t'^* = t' = 0$.

The basic assumption of our experiment is that the one-way speed of light is c in frame S', which thus coincides with the preferred frame of relativistic theories. As justified in the Appendix, if the radius r of the pulley is $r \ll L$, we may omit to consider the time delay corresponding to the motion of O^* on the pulley at the turning point A. Moreover, since the relevant quantities of our thought experiment are of first order in v/c, for simplicity, we neglect higher order terms in our approach.

The rigorous LT between S'' and S' (used in the Appendix) can be obtained from the compositions of the boost from frame S'' to the rest frame S of the arm AB and from S to frame S'. The resulting transformations are, $x' = \gamma_w(x'' - wt'')$; $t' = \gamma_w(t'' - wx''/c^2)$, where $w = 2v/(1 + v^2/c^2)$, and $\gamma_w = (1 - w^2/c^2)^{-1/2}$, and can be approximated by $x' \simeq x'' - wt'' \simeq x'' - 2vt''$; $t' \simeq t'' - wx''/c^2$, where $w \simeq 2v$ and $\gamma_w \simeq 1$.

According to an observer of S'', while the light signal emitted from $O^* \equiv O''^*$ is moving toward point B, point A is moving toward clock O^*. Let us then denote by $t^*_{OA} = t''^*_{OA}$ the time interval measured by clock $O^* \equiv O''^*$ from the moment it emits the signal until the moment point A reaches clock O^*. Then, when A and O^* coincide at the time $t''^* = 0$, we know that the signal has been emitted by O^* at the earlier time $t''^* = -t^*_{OA}$.

According to an observer of frame S', the initial conditions, described without approximations in the Appendix, are such that the light signal (emitted from O^*) reaches point B when clock O^* reaches point A. Thus, the two events: "light signal at B" and "clock $O^* \equiv O'^*$ at A" are simultaneous in S' and the situation is the one reflected by Fig. 1(c) (in the lower section of the belt) at $t''^* = t'^* = 0$. According to S', after covering the path $O^*A = AB \simeq L$ in the belt upper section, at $t''^* = t'^* = 0$ when clock $O^* \equiv O'^*$ is at A, the light signal is at point B and, after being reflected toward A, begins the return trip in the belt lower section. The return trip time measured by clock $O^* \equiv O^{*\prime}$ is denoted by t'^*_{BA} and, then, the round-trip time interval (or proper time) measured by clock O^* can be expressed as

$$t^*_{\text{round}} = t''^*_{O^*A} + t'^*_{BA}, \tag{1}$$

where in Eq. (1) $t^*_{OA} = t''^*_{OA}$ is the outward time interval measured by clock $O^* \equiv O''^*$ and $t^*_{BA} = t'^*_{BA}$ is the return time interval measured by $O^* \equiv O'^*$. Thus, no clock synchronization is needed for the measurement of t^*_{round} performed with the single clock O^*. Obviously, the time interval measured by clock $O^* \equiv O^{*\prime}$ for the return trip of the light signal from B to $O^* \equiv O^{*\prime}$ (located at point A at $t'^* = t' = 0$) must be given by $t'^*_{BA} \simeq L/c$, having assumed that in the inertial frame S' the one-way speed of light is c and that, for SR with absolute synchronization, frame S' coincides with the preferred frame.

As it is well-known,[2] to the first order in v/c and in agreement with the Sagnac effect, the result of our thought experiment is

$$\begin{aligned} t^*_{\text{round}} &= t''^*_{O^*A} + t'^*_{BA}, \\ &\simeq \frac{2L(1 - v/c)}{c} \simeq \frac{2L}{c + v}, \end{aligned} \tag{2}$$

from which, knowing that $t'^*_{BA} \simeq L/c$, we may calculate t''^*_{OA}.

A. Significant difference between the circular and the linear Sagnac effects

In the circular Sagnac effect of Fig. 1(a), the local "ground" path is the circumference of the rotating disk of length $2\pi R$. In the linear Sagnac effect, the total local ground path length of the belt is $2L$ and is represented by the belt upper section of length L when clock O^* is co-moving with S'', and the belt lower section of length L when clock O^* is co-moving with S'. In the circular Sagnac effect, clock O^* is always accelerated because co-moving with a point on the circumference of the rotating disk of radius R. Therefore, the calculation of t^*_{round} in the circular effect has to take into account the delay due to acceleration of clock O^*, which, as pointed out by several authors,[23,24] is not an inertial frame of reference. Instead, in our approach to the linear Sagnac effect, clock O^* is essentially always co-moving with the inertial frame S'' or S'. In this case, the relevant contributions to t^*_{round} are given by t''^*_{OA} and t'^*_{BA}, which reflect measurements made in inertial (nonaccelerated) frames and are in agreement with the observed result $2L/(c + v)$ that is foreseen by theory. Since, as shown in the Appendix, the time delay, due to the accelerated motion of O^* on the pulley at the turning point A is negligible with our approximation, $r \ll L$, our approach using the linear effect removes the difficulties inherent to the accelerated motion of the circular effect.

Furthermore, it is important to stress that the same approximate result t^*_{round} of expression (2) can be derived from the single frame S', S'', S, or any other inertial frame, within Newtonian physics or SR. If the calculations are worked out from a chosen single frame, the path covered by the signal turns out to be $2L(1 - v/c) < 2L$ (as observed from the chosen single frame) because the light signal and the clock O^* are counter-moving. However, the purpose of our thought experiment has nothing to do with the derivation of result (2), but to verify the value of the local speed of light along each section of the path covered by the light signal. In fact, any derivation of result (2) or any interpretation of the Sagnac effect, if worked out from a single frame where the light speed is assumed to be c, is unable to verify the value of the local light speed along every section of the closed light path. Thus, for the experiment of Figs. 1(b) and 1(c), verification of Einstein's second postulate and the local light speed has to be performed in both the upper and lower sections of the belt separately, because the time of flight of the light signal in the corresponding path sections is measured by clock O* and is given by its proper time intervals, first when co-moving with S'' and then with S'. Thus, the local light speed in the belt upper part must correspond to the proper time interval t''^{*}_{OA} measured by clock O* $\equiv$ O$''$* when co-moving with the belt in its upper part. However, the local light speed in the belt lower part must correspond to the proper time interval t'^{*}_{BA} measured by O* $\equiv$ O$'$* in the lower part. Newtonian physics and SR agree that the round trip path to be covered by the signal, starting from O* and back to O*, as shown in Fig. 1(c) is $\simeq L+L=2L$ (the total length of the belt) and, as reflected by result (2), the round trip average speed is $c_{av} \simeq c+v$.

Since, in the return trip, $t'^{*}_{BA} \simeq L/c$, on account of Eq. (2) we have

$$\begin{aligned} t''^{*}_{O*A} &= t^{*}_{\text{round}} + t'^{*}_{BA}, \\ &\simeq \frac{2L}{c+v} - \frac{L}{c} \\ &\simeq \frac{L(1-2v/c)}{c} \simeq \frac{L}{c+2v}, \end{aligned} \tag{3}$$

in agreement with result Eq. (A7) of the Appendix.

III. INTERPRETATION OF LIGHT PROPAGATION IN THE LINEAR SAGNAC EFFECT

At this point, we wish to infer from Eq. (3) the value of the one-way speed of light on the outward path and establish which synchronization, to be adopted in frame S'', is consistent with the results (2) and (3).

A. Absolute simultaneity

Being approximately $2L$ the total path to be covered by the signal, absolute synchronization coherently interprets results (2) and (3). In fact, with the time transformation $t' \simeq t$, reflecting absolute simultaneity, the outward one-way speed in S'' is $dx''/dt = d(x'+2vt')/dt \simeq c+2v$, indicating that absolute simultaneity is in agreement with the superluminal light speed $\simeq c+2v$ of result (3) along the outward path L.

B. Relative simultaneity and the missing light path section

The interpretation of results (2) and (3) by means of Einstein synchronization is the following. In both the outward and return paths, the one-way speed is c (in agreement with Einstein's second postulate) if the length L of the outward path covered by the signal is reduced to $L(1-2v/c) < L$ in Eq. (3). In effect, according to Eq. (3) and as derived in Eq. (A9) of the Appendix, at $t''=0$ the light signal has not yet reached point B but is at the distance O* $D \simeq L(1-2v/c)$ away from O*, i.e., at the point D shown in Fig. 1(c). As a matter of fact, because of nonconservation of simultaneity, the rigorous time transformation $t''=\gamma_w(t'+wx'/c^2)$ indicates that the event "light signal at B" at $t'=0$ for S' occurs at the later time

$$t''_B = \gamma_w \frac{w}{c}\frac{L}{\gamma c} \simeq \delta t'' =\simeq \frac{2vL}{c^2}, \tag{4}$$

for S''. Therefore, at the previous time $t''=0$ in S'', the signal is not yet at the distance $\simeq L$ from O*, but still at the distance O*$D \simeq L - ct''_B \simeq L(1-2v/c)$. Thus, in the case of Einstein synchronization, expression (2) can be written as

$$\begin{aligned} t^{*}_{\text{round}} &= t''^{*}_{O*A} + t''^{*}_{BA}, \\ &= \frac{O*D}{c} + \frac{BA}{c} \\ &\simeq \frac{2L(1-v/c)}{c} \simeq \frac{2L}{c+v}. \end{aligned} \tag{5}$$

Nevertheless, the light signal must have covered also the section DB before covering the return path BO'*. Therefore, expression (5) is missing the time contribution $t''^{*}_{DB} = DB/c = \delta t'' = 2vL/c^2$ corresponding to the missing light path section

$$DB = c\delta t'' \simeq \frac{2vL}{c}. \tag{6}$$

If the missing light path section is taken into account and the corresponding propagation time t''^{*}_{DB} is included in Eq. (2) or Eq. (5), we obtain

$$\begin{aligned} \left(t^{*}_{\text{round}}\right)_E &= t^{*}_{\text{round}} + t''^{*}_{DB} \\ &= \frac{O*D}{c} + \frac{BA}{c} + \frac{DB}{c} \\ &\simeq \frac{2L(1-v/c)}{c} \simeq \frac{2vL}{c^2} \\ &= \frac{2L}{c}, \end{aligned} \tag{7}$$

consistent with Einstein's second postulate requiring that the local light speed be c along the whole path $2L$, but in contrast with experiment.

The absence of the path section DB in Eq. (5) could be justified by assuming, for example, that the corresponding propagation time t''^{*}_{DB} is included in t'^{*}_{BA}. However, in this case, the local speed along this path must be $(\delta t'c+L)/t'^{*}_{BA} \simeq c+2v$, i.e., superluminal. Since result (5) implies the superluminal round-trip average speed $c_{av} \simeq c+v$ over the path $2L$, we conclude that, if the local photon speed is c in one part of the path, in the other part, the local speed must be superluminal and vice versa.

IV. CONCLUSIONS

Our thought experiment indicates that the outcome of the Sagnac effect favors absolute simultaneity, which allows for superluminal light speeds and introduces no missing light paths. In formulating SR, Einstein has the merit of having introduced the relativistic effects of time dilation and length contraction, inherent to the LT. These effects are what gives physical meaning to SR and makes possible the interpretation of the known experiments supporting SR, with either absolute or relative simultaneity.[1] However, in the interpretation of the Sagnac effect, the postulate on the constancy of the speed of light (and relativity of simultaneity) appears to be inconsistent with an average superluminal speed $c_{av} \simeq c + v$ along the path $2L$ and introduces a missing light path that cannot be justified by the observer co-moving with clock O*. The type of synchronization adopted by a theory should reflect and preserve the uniqueness of physical reality. Relativity of simultaneity can be objected to because, as shown in Fig. 1(c), it requires that, at the time $t^* = t''^* = t'^* = 0$, an observer at point A, depending on its velocity, sees the light signal being located at the distance $O''^*D = L(1-2v/c)$ or $O'^*B = L$ from $O''^* \equiv O'^*$, as if the signal could jump from D to B in no time, violating physical determinism and the uniqueness of physical reality. Any attempt to justify relative simultaneity will not modify the fact that absolute simultaneity provides a simpler and more coherent way to interpret the linear Sagnac effect. To recover consistency, the time paradigm of SR needs to reinstate conservation of simultaneity, even though allowing for time dilation. Certainly, SR is capable of interpreting all of the experiments supporting the theory by adopting either absolute or relative simultaneity. However, according to Ref. 19, absolute and relative simultaneity can be discriminated experimentally and, thus, only one of the two will be corroborated by observation. The fact that no paradoxes or inconsistencies are found when absolute simultaneity is adopted,[1,2] suggests that absolute synchronization is a viable alternative for SR. In effect, the feat of Ref. 19 indicates that, if the theories and models developed with the LT are also elaborated with transformations based on absolute simultaneity and the results are compared, most likely the two approaches will manifest meaningful observable differences, thus leading to new insights and a better comprehension of SR and natural phenomena.

The required change from Einstein to absolute synchronization has important implications in the several areas of modern physics based on the LT, such as electrodynamics, elementary particles, cosmology, and field theories, just to name a few. The issue of implications requires a detailed discussion that we will leave to future contributions. However, we mention here, to begin with, that absolute simultaneity requires the existence of a preferred frame where the one-way speed of light is c. Several authors have suggested that such a frame is the Earth Centered Inertial (ECI) frame.[15,16,20,25] We notice that, in order to reconcile the observable result (2) of the Sagnac effect with result (7) (consistent with Einstein's second postulate), the latter must be corrected by the missing time delay $\delta t''$. According to some authors,[15,16,20,25] in the context of the ECI frame, similar situations emerge for the clock synchronization in the Global Positioning System (GPS). In fact, in order to maintain accuracy, the Global Positioning System must apply a Sagnac velocity correction to its electromagnetic signals. Once performed, experiments that can discriminate absolute from relative simultaneity[19] can corroborate (or not) this or similar points of view. Other experiment reviews, like those by Ahmed,[26] validate the idea that a substantial number of members in the scientific community are pursuing similar objectives.

It is worth mentioning that the standard gravity theory also predicts that the speed of gravity is the same as the speed of light.[27,28] Recent experiments seem to have confirmed that the speed of gravity is approximately the speed of light,[29–33] but the measurement errors are much larger than those seen in traditional speed of light experiments. Our finding that Lorentz symmetry breaks down in a linear Sagnac setup could possibly even have implications for gravity. Several quantum gravity theories suggest that Lorentz symmetry could be broken, so there may be a direct link here. Still, whether or not our work is directly relevant in relation to gravity, it is too early to say. Naturally, the lack of a broadly accepted quantum gravity theory does not make it easier. However, our new research is clearly relevant for several fields in physics and will hopefully inspire more research along these lines.

APPENDIX: ADDITIONAL INFORMATION

1. Time delay at the clock turning point A

In the linear Sagnac experiment,[22] clock O* changes direction at the pulley turning point A while moving with velocity v relatively to frame S. Therefore, there is a time delay τ experienced by clock O*, in part due to its motion on the circumference of the pulley of radius r, and in part because it has been accelerated in passing from S'' to S'. The delay $\tau \sim r/v$, corresponding to the motion of O* around the pulley depends on r and v, while the acceleration $a \simeq v^2/r$, which modifies the rate of time as measured by clock O*, depends on v and the short path $\sim r$ along which it is acting. According to the principle of equivalence, the resulting time change due to a is proportional to the gravitational potential change ar, but does not depend on the path length $2L$. Then, in any case, for our thought experiment, τ does not depend on L and we may assume L to be large enough for τ to be negligible in comparison to any time delay proportional to L.

According to standard SR, the time delay $\delta t'' \simeq 2Lv/c^2$ due to nonconservation of simultaneity is a real effect and, although it is unclear how to do it, should be observable. Going beyond the use of the conveyor belt system as a thought experiment, if we wish to perform in practice a real experiment capable of corroborating the theoretical result of Eq. (2), $t^*_{\text{round}} \simeq 2L/(c+v)$, while having the time delay τ negligible relative to the delay $\delta t''$, the length L of the conveyor belt arm AB has to be quite large and can be estimated as follows. If $\tau \sim r/v \simeq 2Lv/c^2 \simeq \delta t''$, we find $L \simeq rc^2/v^2$. With $r = 10^{-1}$ m and $v = 300$ m/s, we have $L \simeq 10^{11}$ m, which is more or less the distance between Earth and Sun. Thus, it is obvious that physical effects associated with nonconservation of simultaneity are very small and unlikely to be detectable, even if were observable in principle. However, it is not inconceivable to expect, in some elaborated experimental setup of this type, that the delay $\delta t''$ can be accumulated at each cycle of the light signal around the path $2L$. In this case, the length L could be reduced to more manageable values.

2. Initial conditions

As indicated in Figs. 1(b) and 1(c), the inertial frame S' is co-moving with the lower section of the belt, while clock O* is the origin of the inertial frame S'', co-moving with the upper section of the belt, which is moving with velocity $-w$ relative to S'. We wish to determine the distance of point A from clock O* when the light signal is emitted. The initial condition to be imposed must be such that the light signal reaches point B when clock O* reaches point A, as evaluated simultaneously from frame S'.

We assume here that, as shown in Fig. 1(b), at the epoch $t' = t'' = 0$, clock O* (the origin of frame S'') coincides with the origin of S' and, relative to frame S', point A is at $x_A'(0) = -L_0/\gamma$ and point B at $x_B'(0) = (L - L_0)/\gamma$ being $x_B'(0) - x_A'(0) = L/\gamma$. In fact, the conveyor arm $AB = L$, at rest in frame S and moving with velocity v relative to S', is contracted by the factor $\gamma = (1 - v^2/c^2)^{-1/2}$. The relative velocity w between S' and S'' is determined by the assumption that speed of the belt sections is v relative to the conveyor arm AB. Therefore, the velocity w of frame S'(co-moving with the lower section of the belt), relative to frame S'' (co-moving with the upper section of the belt) is given by the relativistic composition of the velocity v of S' relative to S, and the velocity v of S relative to S''. The standard relativistic addition of velocities formula is $u'' = (u + v)/(1 + vu/c^2)$, where u'' is the velocity of a point particle relative to S'' and u is the corresponding velocity relative to S. Since, in our case, $u'' = w$ and $u = v$, we find $w = (v + v)/(1 + vv/c^2)$, i.e.,

$$w = \frac{2v}{1 + v^2/c^2}. \tag{A1}$$

Thus, the LT between S'' and S' are, $x' = \gamma_w(x'' - wt'')$; $t' = \gamma_w(t'' - wx''/c^2)$, where $\gamma_w = (1 - w^2/c^2)^{-1/2} = \gamma^2(1 + v^2/c^2)$.

In frame S', the equation of motion of point A is, $x_A'(t') = x_{0A}' - vt' = -L_0/\gamma - vt'$, while that of O* is $x_O^{*'}(t') = -wt'$. Thus, from the equation $x_A'(t') = x_O^{*'}(t')$, we have, $-L_0/\gamma - vt' = -wt'$, and we find that A and O* meet at the time

$$t' = t_A' = \frac{L_0}{\gamma(w - v)}. \tag{A2}$$

Analogously, for point B we have, $x_B'(t') = x_{0B}' - vt' = (L - L_0)/\gamma - vt'$ and, for the light signal $x_c'(t') = ct'$. The equation determining the time when the light signal reaches B is, $x_B'(t') = x_c'(t')$ or, $(L - L_0)/\gamma - vt' = ct'$, which gives,

$$t' = t_B' = \frac{L - L_0}{\gamma(c + v)}. \tag{A3}$$

By requiring that the two events "clock O* at A" and "light signal at B" are simultaneous in frame S' with the help of Eqs. (A2) and (A3) the condition $t_A' = t_B'$ gives

$$\frac{L_0}{\gamma(w - v)} = \frac{L - L_0}{\gamma(c + v)}. \tag{A4}$$

By substituting in Eq. (A4) the value of w given in Eq. (A1), after some algebra we find

$$L_0 = L\frac{v}{c}\frac{1 - v/c}{1 + v/c}, \tag{A5}$$

$$t' = t_B' = \frac{L}{c}\frac{\gamma_w}{\gamma}\frac{1 - v/c}{1 + v/c}. \tag{A6}$$

By means of the time transformation $t' = \gamma_w(t'' + wx''/c^2)$ with $x_O{}^{*''} = 0$, evaluated in S'', the time interval taken by A to reach O* is

$$t''_{AO^*} = \frac{t'A}{\gamma_w} = \frac{L}{c\gamma}\frac{1 - v/c}{1 + v/c} \simeq \frac{L(1 - 2v/c)}{c}. \tag{A7}$$

Therefore, since A is has been moving with velocity v relatively to O*, at $t'' = 0$ in frame S'', the distance AO* is

$$AO* = vt''_{AO} = v\frac{t'^A}{\gamma_w} = L\frac{v}{c}\frac{1 - v/c}{\gamma(1 + v/c)} \simeq L\frac{v}{c}. \tag{A8}$$

During the time interval $t_{AO}{}^{*''}$, according to frame S'', the light signal has covered the distance

$$O^*D = AD = ct''_{AO*} = L\frac{1 - v/c}{\gamma(1 + v/c)} \simeq L(1 - 2v/c). \tag{A9}$$

Once the initial conditions have been established and t''^*_{OA} and $O^*D = AD$ have been calculated, it is convenient to describe the propagation of the light signal by means of frames S' and S'' having their origins coinciding with point A, as indicated in Fig. 1(c). Then, for simplicity, we reset the clocks O*, O''* and O'* to $t'' = t' = 0$, which is equivalent to reset $t''^*_{OA} \Rightarrow -t''^*_{OA}$, performing a simple time translation. This way, we may now calculate the return time of the light signal, $t'^*_{BA} \simeq L/c$, as done for Eq. (2).

3. The effect of nonconservation of simultaneity

According to the authors of Refs. 2 and 16–18, Einstein synchronization fails when performed along a closed path. Given that the path covered by the light signal is $2L$, the result consistent with Einstein's second postulate about the constancy of the speed of light c is $(t^*_{\text{round}})_E = 2L/c$, as in Eq. (7). However, expression (7) does not agree with the result of the Sagnac effect. Nevertheless, by introducing relative simultaneity, the LT introduce the time difference $\Delta t''_{AB} \simeq 2vL/c^2$, given in Eq. (4), between the events light signal at B and clock $O^* \equiv O'^*$ at A. This way, at $t'' = 0$, the time taken by the signal to cover the distance $AB = L$ in the upper section of the belt is reduced to $L/c - 2vL/c^2 = (L/c)(1 - 2v/c) = O^*D/c$, as in Eq. (5). Yet, in this case, the mentioned inconsistency regarding the missing path DB is introduced. This inconsistency is thus related to the failure of Einstein synchronization when performed along a closed path.

[1]R. Mansouri and R. U. Sexl, Gen. Relativ. Gravitation **8**, 497 (1977).
[2]K. Kassner, Am. J. Phys. **80**, 1061 (2012).
[3]G. Sagnac, C. R. Acad. Sci. **157**, 708 (1913).
[4]H. Poincaré, Bull. Sci. Math. **28**, 302 (1904).
[5]A. Einstein, Ann. Phys. **17**, 891 (1905).
[6]R. Reichenbach, *Axiomatization of the Theory of Relativity*, 1st German ed. 1924 (University of California Press, Berkeley, CA, 1969).
[7]A. Grünbaum, *Philosophical Problems of Space and Time* (Alfred A. Knopf, New York, 1963).
[8]F. R. Tangherlini, Nuovo Cimento Supp. **20**, 1 (1961).
[9]R. de Abreu and V. Guerra, Eur. J. Phys. **26**, 117 (2005); Found. Phys. **36**, 1826 (2006); Eur. J. Phys. **29**, 33 (2008).
[10]J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, UK, 1988).
[11]R. Anderson, I. Vetharaniam, and G. E. Stedman, Phys. Rep. **295**, 93 (1998).
[12]H. C. Ohanian, Am. J. Phys. **72**, 141 (2004).
[13]G. Spavieri, Eur. Phys. J. D **66**, 76 (2012).
[14]G. Spavieri, J. Quintero, C. S. Unnikrishnan, G. T. Gillies, G. Cavalleri, E. Tonni, and L. Bosi, Phys. Lett. A **376**, 795 (2012).
[15]F. Selleri, Found. Phys. Lett. **10**, 73 (1997); Apeiron **11**, 246 (2004).
[16]S. J. G. Gift, Appl. Phys. Res. **7**, 112 (2015).
[17]T. A. Weber, Am. J. Phys. **65**, 946 (1997).
[18]J. Anandan, Phys. Rev. D **24**, 338 (1981).
[19]G. Spavieri, M. Rodriguez, and A. Sanchez, J. Phys. Commun. **2**, 085009 (2018).
[20]J. H. Field, Fund. J. Mod. Phys. **10**, 1 (2017); **11**, 1 (2018); **11**, 135 (2018).
[21]R. Lundberg, Phys. Essays **32**, 237 (2019).
[22]R. Wang, Y. Zhengb, A. Yaob, and D. Langley, Phys. Lett. A **312**, 7 (2003).
[23]E. J. Post, Rev. Mod. Phys. **39**, 475 (1967).
[24]G. B. Malykin, Phys.-Usp. **45**, 907 (2002).
[25]D. W. Allan and M. A. Weiss, Science **228**, 69 (1985).
[26]F. Ahmed, B. M. Quine, S. Sargoytchev, and A. D. Stauffer, Indian J. Phys., **86**, 835 (2012).
[27]S. M. Kopeikin and E. B. Fomalont, Found. Phys. **36**, 1244 (2006).
[28]W. W. Salisbury, Nature **224**, 782 (1969).
[29]S. Krasnikov, Gravit. Cosmol. **17**, 194 (2011).
[30]S. Du, F. Peng, and M. Li, e-print arXiv:1712.05965 (2017).
[31]N. Cornish, D. Blas, and G. Nardini, Phys. Rev. Lett. **119**, 161102 (2017).
[32]O. J. Tattersall, P. G. Ferreira, and M. Lagos, Phys. Rev. D **97**, 084005 (2018).
[33]E. G. Haug, Int. J. Astron. Astrophys. **9**, 97 (2019).