Space-Time "Pareto Optimality"
Arbitrage-free Space-Time Equilibrium

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This is a very short and ugly draft-note on space-time Pareto optimality, that is a type of arbitrage-free space-time equilibrium. The analysis is based on Einstein's Special Relativity Theory (1905) combined with general theories in economics.

I am not sure Pareto optimality is good term in this context; it is a long time since I went to business school reading about Pareto optimality. My point is that there under some idealistic assumptions that do not interfere directly with relativity theory there will exist a type of economic equilibrium where there actually is no space-time arbitrage without leaving Pareto optimality. That is if someone are going to arbitrage then some others will be worse off because the arbitrage. And rational people (under very idealistic assumptions) will not accept to get it worse because someone else are arbitraging them, so if no transaction costs in peoples movement etc. there will exist a theoretical equilibrium where there are no basic interest rate space-time arbitrage.

The clue is to see that in long run real interest rate returns must be related to production and then to link this to space-time.

If someone leave earth in a high velocity spaceship they can initially make interest rate arbitrage, see Haug (2002, 2004). For simplicity assume the spaceship is leaving earth and returning to earth. This makes it simpler to know exactly which observer moved faster relative to the other. In this case the spaceship will clearly be the moving observer and the earth the stationary observer.

Idealistic assumptions:
- Anyone can move between the spaceship and earth, as they want without transaction costs. Naturally only when it docs at earth, for example at the end of each earth year. Alternatively it could rotate just outside earth, with transportation of people going back and forth, but this would naturally make the calculations more complicated without giving more insight on the main principle of idealistic space-time arbitrage free equilibrium.
- People on earth and in the spaceship have the same production capacity per hour of wristwatch time.
- Production is linearly related to human input and time. 4 humans can produce twice as much as 2 in same amount of wristwatch time (local time).

[1] Based on an idea I came up with in 2002/2003 and that I had in an early draft of my SpaceTime Finance paper.

- Production is linearly related to wristwatch time. 2 humans can produce twice as much as 2 other humans if twice the wristwatch time.
- Same access to production resources on the space ship and earth (for moving and stationary observer).

In general interest rats can only be paid if there is production (look away from inflating money supply etc.) and relative to the size of the production. Zero production gives zero return. Twice the production gives twice the return. Interest rates paid because of inflating money supply is in general and under simplified assumptions just compensation for lower purchasing power and is excluded in this analysis. We are thinking about real returns that in general and under some simplified assumptions are directly related to production.

Now assume the early stage of high velocity spaceships. Some people leave earth to live on the spaceship. They travel at velocity that makes 1-year wristwatch time goes by for every 2-year wristwatch time on earth. By placing their money on earth they get much higher return than if they placed their money in the bank on the spaceship, even if the bank on spaceship paid the same interest rate. This is simply due to time is moving faster on earth than on the spaceship.

However since time moves much slower on the high velocity spaceship relative to the time on earth the production on the spaceship must also be lower under these assumptions. This will mean banks on the spaceship (lending out their money to production on the spaceship) not can give the same rate of return as banks based on earth (lending out their money to production on earth). Now if free mobility of people and capital, everyone would want to move to the spaceship and leave their money in the banks on earth. Then however production on earth would collapse because the production was assumed to be linearly dependent on human input.

By simple calculations we can find a space-time arbitrage free equilibrium (Pareto Optimality) that we would get under these idealistic assumptions (assuming also zero fuel cost I guess). Pareto Optimal space-time equilibrium we would get when no one would get it better by moving a person from the earth to the spaceship of vice versa without someone getting worse off.

Lots of different scenarios based on different assumptions can here be made, where production not is linear relative to people input, for example with use of robots, and where we take into account the fuel cost and the cost of building and maintaining spaceships[2]. For example if robots could do all production, then all humans could improve their returns (and living standards) relative to their wristwatch time by space-time arbitrage, placing their money on earth, living on the high velocity spaceship. Basically they are then arbitraging the robots that I guess must have lower IQ (or at least restriction of movements), if not they would move to the spaceship also, and if higher IQ they would move to spaceship and force the humans to produce for them on earth (potentially the most likely future scenario ;-). In an advanced space travel age we would

[2] See Hickman (1999) for an article on "The Political Economy of Very Large Space Projects".

naturally also need to take into account interstellar tax rules, see Chodorow (2009).

Conclusion:

Under idealistic assumptions there are no free lunch also in space and time. But in emerging markets, and an emerging age of space travel there will be plenty of good arbitrage opportunities. Bring it On!

References

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